ICPC 2018–2019 NEERC – Northern Eurasia Finals Problems Review

December 2, 2018

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> These slides give only brief idea of general solution direction

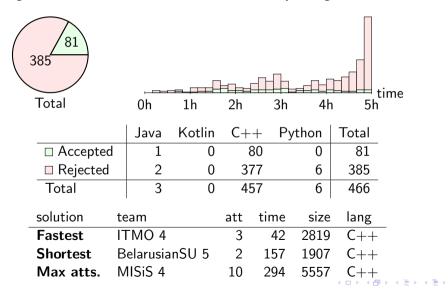
- Recap: 299 teams, 13 problems, 5 hours
- Full text analysis and problem statements are published at http://neerc.ifmo.ru/
- These slides give only brief idea of general solution direction
- Summary table on the next slide lists problem name and stats
 - acc number of teams that had solved the problem (gray bar denotes a fraction of the teams that solved the problem)
 - runs number of total attempts
 - succ overall successful attempts rate (percent of accepted submissions to total, also shown as a bar)

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| problem name | acc/runs | succ |
|----------------------------|----------|------|
| Alice the Fan | 81 /466 | 17% |
| Bimatching | 0 /53 | 0% |
| Cactus Search | 26 /121 | 21% |
| Distance Sum | 0 /19 | 0% |
| Easy Chess | 249 /565 | 44% |
| Fractions | 148 /677 | 21% |
| Guest Student | 225 /589 | 38% |
| Harder Satisfiability | 1/11 | 9% |
| Interval-Free Permutations | 2 /5 | 40% |
| JS Minification | 5 /50 | 10% |
| King Kog's Reception | 20 /65 | 30% |
| Lazyland | 247 /490 | 50% |
| Minegraphed | 66 /278 | 23% |

Author: Oleg Hristenko

Statements and tests: Niyaz Nigmatullin



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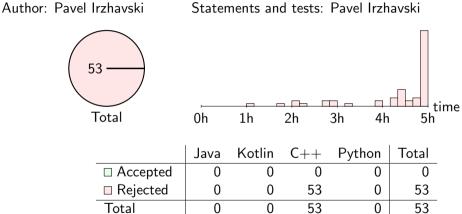
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- There are $O(C^2)$ states $(5 \cdot C \cdot C)$.
- Each set has O(C) possible outcomes.
- ► To restore the answer compute f_{r,i,j} the outcome of the r-th set that results in optimal answer for d_{r,i,j}.



Statements and tests: Pavel Irzhavski

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- > Put all nodes of original graph in a right part and middle points in a left part.
- ▶ Now, solving bimatching problem solves maximum matching, thus it is not easier.
- As an experienced participant you should stop to solve the problem with maximum flow or minimum cost maximum flow and think whether you can convert this two problems in the other direction and solve bimatching with maximum matching.

• For each node v in the left part create its clone v'.

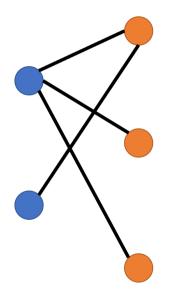
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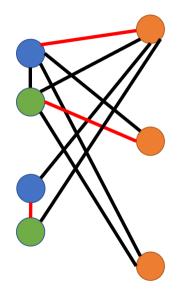
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- Now, add an edge between v and its clone v'.
- ▶ Find maximum matching in this new graph. Its size is at least *n* (size of the left part).
- ► However, switching v and v' to match with some nodes in the right part increases the answer by 1.





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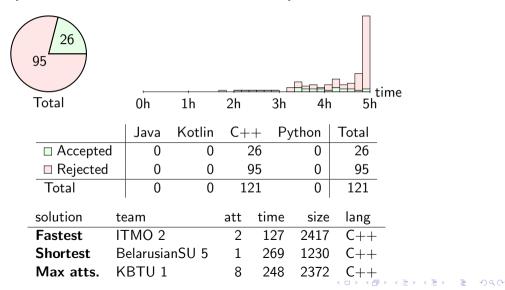
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- ► To find whether there exists a perfect matching in the graph one checks whether $|M| \neq 0$.
- To find the size of the maximum matching we apply Gauss elimination to find rank of Tutte matrix.
- FYI, the best known result to find maximum matching is $O(\frac{VE}{\log V})$.

Problem C. Cactus Search

Author: Borys Minaiev

Statements and tests: Borys Minaiev



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- Each tree has at least one centroid, such node that its removal splits the tree in connected components of size no more than ⁿ/₂.

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- The process will finish in no more than $\log n + 1$ steps.

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- Now we consider cactus case.
- Actually, ignore the fact we are given a cactus and solve the problem for any connected graph.
- Apply Floyd algorithm to compute a matrix of distances $\rho(u, v)$.
- We maintain a set of candidates A. Initially it contains all nodes of given graph G.
- If we ask about node u and node v is given as a reply we eliminate from set A all nodes w ∈ A such that ρ(v, w) < ρ(u, w).</p>

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- We aim to eliminate as many nodes from A as possible.
- Consider every node v (not necessaritly in A). For each u neighbor of v compute the number of nodes w in A that are closer to u.
- Pick v that has minimum maximum number of such w's among all u's.
- ▶ How many steps it will take to reduce A down to a single node?
- We claim there always exists such v that the size of A will reduce at least twice.

Problem C. Cactus Search

Let P(v) = ∑_{u∈A} ρ(v, u). If more than a half of nodes w ∈ A are closer to some u (neighbor of v), then P(u) < P(v).</p>

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• That results in $O(n^3)$ per test case.

Author: Gennady Korotkevich Statements and tests: Gennady Korotkevich 19 time Total 1h2h 3h 4h 5h 0h Java Kotlin Python Total C++ \Box Accepted 0 0 0 0 0 Rejected 2 0 8 9 19 8 Total 2 0 9 19

• Need:
$$\sum_{u=1}^{n-1} \sum_{v=u+1}^{n} d(u, v)$$

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• Need:
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• Generalize! Vertex
$$i \rightarrow$$
 weight w_i

• Need:
$$\sum_{u=1}^{n-1} \sum_{v=u+1}^{n} w_u w_v d(u, v)$$

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▶ While there is a vertex of degree 1:

• Pick any such vertex v, let its only neighbor be u

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- Add $w_v \cdot (n w_v)$ to the answer
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- If there is a single vertex of degree 0, exit
- Otherwise all vertices have degree at least 2
- $\deg(v) > 2 \implies v$ is special
- ▶ $m \le n + 42 \implies$ there are at most 84 special vertices

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- > 84 special vertices connected by paths of non-special vertices
- > Find the distance from every special vertex to all other vertices using BFS

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•
$$s(v) = \sum_{u \neq v} w_u d(u, v)$$

• The answer is $\frac{1}{2} \sum_{v} s(v)$

• How to calculate s(v)?

- ► How to calculate *s*(*v*)?
- For special vertices u, d(u, v) is already known

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 - $u_1, x_1, x_2, \ldots, x_k, u_2$
- The shortest path from v to u visits either u_1 or u_2
 - ► $\exists t \in [0; k]$: the shortest path from v to x_1, x_2, \ldots, x_t visits u_1 , and the shortest path from v to $x_{t+1}, x_{t+2}, \ldots, x_k$ visits u_2

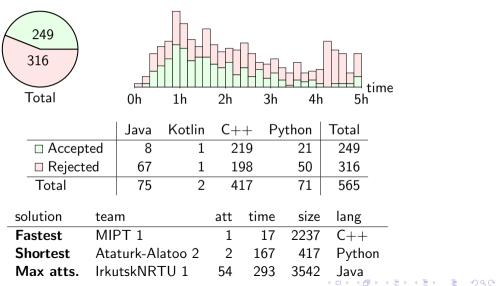
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- ▶ t can be calculated based on $d(v, u_1)$, $d(v, u_2)$, and k
- ► The part of s(v) dependent on x₁, x₂,..., x_k can be calculated based on prefix sums of w_{x1}, w_{x2},..., w_{xk} and 1 · w_{x1}, 2 · w_{x2},..., k · w_{xk}

Author: Mikhail Dvorkin

Statements and tests: Mikhail Dvorkin



- ▶ Visited cells per column, *n* = 2: [1, 0, 0, 0, 0, 0, 0, 2].
- Add +1 arbitrarily n 2 times, keeping ≤ 8 .

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- Process columns left to right. If visited[i] is 0: skip.
- > Otherwise: enter at current row, arbitrarily visit some cells.

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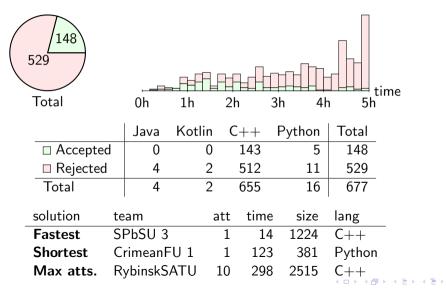
- Constraints:
 - ▶ in column a: start in row 1
 - in columns a—g: don't finish in row 8
 - ▶ in column h: finish in row 8

Problem F. Fractions

Author: Dmitry Yakutov

Statements and tests: Dmitry Yakutov

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Problem F. Fractions

- ▶ If $n = p^k$, where k > 0 and p is prime, then print "NO".
- Otherwise there exist such integers a, b that $n = a \cdot b$, 1 < a, b < n, gcd(a, b) = 1, $a \le \sqrt{n}$. You can do it in $O(\sqrt{n})$.
- Let's try to represent $\frac{n-1}{n}$ as a sum of two fractions $\frac{n-1}{n} = \frac{x}{a} + \frac{y}{b}$.
- It is always possible to find such x and y $(1 \le x < a, 1 \le y < b)$ that $\frac{n-1}{n} = \frac{x}{a} + \frac{y}{b}$.
- ▶ Just iterate over all possible values x between 1 and a 1 and check that $y = \frac{n-1-x \cdot b}{a}$ is integer. It requires $O(\sqrt{n})$ steps.
- Total time complexity is $O(\sqrt{n})$.

Problem F. Fractions: Proof

- For each x between 1 and a 1 the value $y = \frac{n-1-x \cdot b}{a} > 0$ (since $a \cdot b = n$).
- Show that $y = \frac{n-1-x \cdot b}{a}$ is integer for some x between 1 and a-1.
- Let's try all $x = 0 \dots a 1$ and look on $(n 1 x \cdot b) \mod a$.

• For
$$x = 0$$
 $(n - 1 - 0 \cdot b) \mod a \neq 0$.

▶ If for all $x = 1 \dots a - 1$ all values $(n - 1 - x \cdot b) \mod a \neq 0$, then there are two equal remainder.

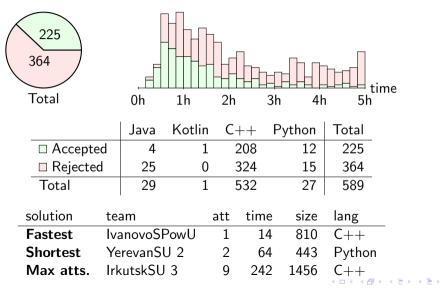
- Say, $(n 1 x_1 \cdot b) \mod a = (n 1 x_2 \cdot b) \mod a$.
- ▶ It means $(x_1 x_2) \cdot b \mod a = 0$, but it is imposible since $|x_1 x_2| < a$.

Problem G. Guest Student

Author: Mikhail Mirzayanov

Statements and tests: Mikhail Mirzayanov

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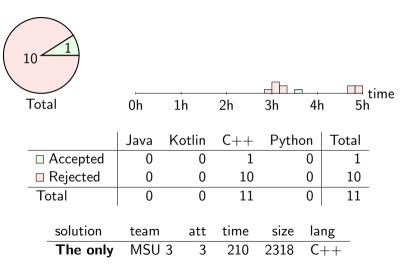


- Try each day of a week to start education.
- ▶ For each fixed starting day of week solve the problem independently.
- ► Calculate number of whole weeks. Roughly speaking, number of whole weeks is $\approx k/(a_1 + a_2 + \cdots + a_7)$. Process the remainder day by day.

▶ Return the best result over all possible 7 starting days of a week.

Problem H. Harder Satisfiability

Author: Andrey Stankevich Statements and tests: Artem Vasilyev



Recall a solution for classic 2-SAT problem:

- Convert formula to implication graph with vertices x_i, x̄_i for all variables: x ∨ y ⇒ (x̄ → y), (ȳ → x)
- Important property: if x is reachable from y, than \overline{y} is reachable from \overline{x} .

- ▶ Find strong connected components (SCC) in this graph.
- If for any vertex x and \overline{x} in same component, no solution exists.
- Otherwise, assign values to variables using the topological order.

Necessary conditions:

- For all *i*: x_i and $\overline{x_i}$ are not in same SCC.
- For all i < j with ∃ quantifier near x_i and ∀ near x_j: none of x_i, x_i is in one SCC with x_j, x_j.
- For all *i*, *j* with ∀ quantifiers: none of x_i, x_i, x_j, x_j are not reachable from each other.

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Problem H. Harder Satisfiability

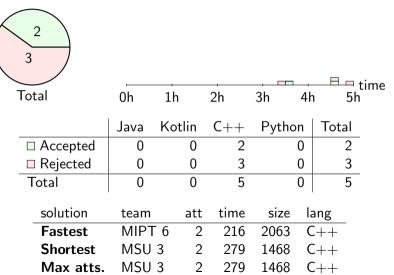
And they are sufficient!

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- For all *i*, *j* with ∀ quantifiers: none of x_i, x_i, x_j, x_j are not reachable from each other.

Assign algorithm:

- Condition 2 allows us to mark SCC as "any", if it have a \forall vertex inside.
- Let's make all components reachable from "any" as true, and their negations as false.
- This doesn't lead to a contradiction, because if some vertex is reachable from one *forall* and some *forall* is reachable from it, then the third condition fails.
- Other components can be decided the same way as classic 2-SAT

Author: Andrey Stankevich Statements and tests: Pavel Kunyavsky



- Use dynamic programming. Count all permutations and subtract bad ones
- Let's call an interval a block, if it's not inside any other interval, except for an entire permutation.
- ▶ Two blocks either non-intersecting or cover full permutation

- Permutation is either split by at least three blocks or covered by two.
- In the first case, splitting is unique. We can choose any permutation in each block, while permutation of blocks should be good.
- In the second case, there are several ways to split. To make it unique, we need to force left permutation have no prefixes, which are permutation.

• Number of permutation, none of prefixes of which is permutation. $I_n = n! - \sum_{k=1}^{n-1} I_k \cdot (n-k)!$

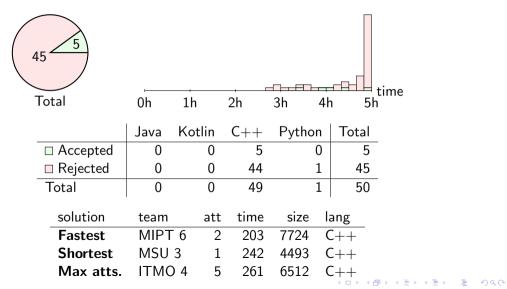
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- ► Number of ways to choose *k* permutations of total length *n*. $B_{n,k} = \sum_{t=1}^{n} B_{n-t,k-1} \cdot t!$
- The answer

$$A_n = n! - 2 \cdot \sum_{k=1}^{n-1} I_k \cdot (n-k)! - \sum_{k=3}^{n-1} B_{n,k} \cdot A_k$$

Author: Roman Elizarov

Statements and tests: Roman Elizarov



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 - Either longest word/number or a reserved token

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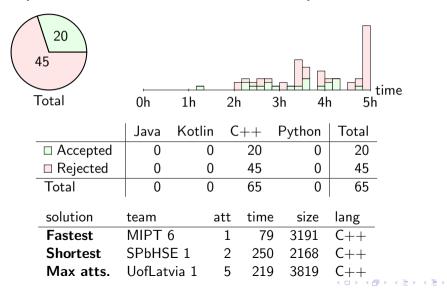
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- Rename words
 - Remember to skip reserved

- Parse input
 - Either longest word/number or a reserved token
- Rename words
 - Remember to skip reserved
- Write output
 - Greedily insert spaces when needed
 - Keep a list of tokens written since the last space

Author: Vitaliy Aksenov

Statements and tests: Vitaliy Aksenov



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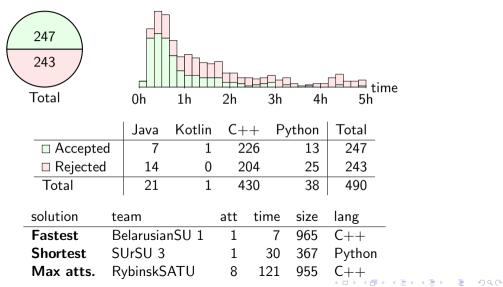
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- Right part: interval tree with sum
- Left part: adding/removing a knight affect a segment.
- So, interval tree with addition on a segment.

Problem L. Lazyland

Author: Pavel Mavrin

Statements and tests: Pavel Mavrin



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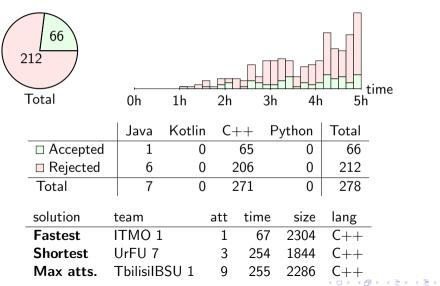
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Assign idlers with minimum values to remaining jobs.

Author: Mikhail Dvorkin

Statements and tests: Mikhail Dvorkin



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- ▶ For each *i*, make 2-step "staircase" from bottom to top.
- For each edge i → j, make vertical hole to fall from i-th top tunnel to j-th bottom tunnel.

Credits

 Special thanks to all jury members and assistants (in alphabetic order):

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