# ICPC 2018-2019 <br> NEERC - Northern Eurasia Finals <br> Problems Review 

December 2, 2018

## Problems summary

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- Summary table on the next slide lists problem name and stats
- acc - number of teams that had solved the problem (gray bar denotes a fraction of the teams that solved the problem)
- runs - number of total attempts
- succ - overall successful attempts rate (percent of accepted submissions to total, also shown as a bar)


## Problems summary

| problem name | acc/runs | succ |
| :--- | :---: | ---: |
| Alice the Fan | $81 / 466$ | $17 \%$ |
| Bimatching | $0 / 53$ | $0 \%$ |
| Cactus Search | $26 / 121$ | $21 \%$ |
| Distance Sum | $0 / 19$ | $0 \%$ |
| Easy Chess | $249 / 565$ | $44 \%$ |
| Fractions | $148 / 677$ | $21 \%$ |
| Guest Student | $225 / 589$ | $38 \%$ |
| Harder Satisfiability | $1 / 11$ | $9 \%$ |
| Interval-Free Permutations | $2 / 5$ | $40 \%$ |
| JS Minification | $5 / 50$ | $10 \%$ |
| King Kog's Reception | $20 / 65$ | $30 \%$ |
| Lazyland | $247 / 490$ | $50 \%$ |
| Minegraphed | $66 / 278$ | $23 \%$ |

## Problem A. Alice the Fan

Author: Oleg Hristenko Statements and tests: Niyaz Nigmatullin


|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 1 | 0 | 80 | 0 | 81 |
| $\square$ Rejected | 2 | 0 | 377 | 6 | 385 |
| Total | 3 | 0 | 457 | 6 | 466 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | ITMO 4 | 3 | 42 | 2819 | C++ |
| Shortest | BelarusianSU 5 | 2 | 157 | 1907 | C ++ |
| Max atts. | MISiS 4 | 10 | 294 | 5557 | C++ |

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- There are $O\left(C^{2}\right)$ states $(5 \cdot C \cdot C)$.
- Each set has $O(C)$ possible outcomes.
- To restore the answer compute $f_{r, i, j}$ - the outcome of the $r$-th set that results in optimal answer for $d_{r, i, j}$.


## Problem B. Bimatching

Author: Pavel Irzhavski

Statements and tests: Pavel Irzhavski


|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 0 | 0 | 0 |
| $\square$ Rejected | 0 | 0 | 53 | 0 | 53 |
| Total | 0 | 0 | 53 | 0 | 53 |

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- Now, solving bimatching problem solves maximum matching, thus it is not easier.


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- First we observe that bimatching as not easier than maximum matching in general graphs.
- Indeed, consider some graph. You can split each of its edges in the middle and add extra node there.
- Put all nodes of original graph in a right part and middle points in a left part.
- Now, solving bimatching problem solves maximum matching, thus it is not easier.
- As an experienced participant you should stop to solve the problem with maximum flow or minimum cost maximum flow and think whether you can convert this two problems in the other direction and solve bimatching with maximum matching.


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- Now, add an edge between $v$ and its clone $v^{\prime}$.
- Find maximum matching in this new graph. Its size is at least $n$ (size of the left part).
- However, switching $v$ and $v^{\prime}$ to match with some nodes in the right part increases the answer by 1 .




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- To find the size of the maximum matching we apply Gauss elimination to find rank of Tutte matrix.


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- However, assuming the given constraints there is no sense to aim for $O(V E)$ and one can go with easy to implement $O\left(V^{2} E\right)$ version.
- Another method to approach maximum matching problem is to consider Tutte matrix $M$ of this graph.
- To find whether there exists a perfect matching in the graph one checks whether $|M| \neq 0$.
- To find the size of the maximum matching we apply Gauss elimination to find rank of Tutte matrix.
- FYI, the best known result to find maximum matching is $O\left(\frac{V E}{\log V}\right)$.


## Problem C. Cactus Search

Author: Borys Minaiev
Statements and tests: Borys Minaiev


|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 26 | 0 | 26 |
| $\square$ Rejected | 0 | 0 | 95 | 0 | 95 |
| Total | 0 | 0 | 121 | 0 | 121 |
| solution | team |  | att | time | size |
| lang |  |  |  |  |  |
| Fastest | ITMO 2 | 2 | 127 | 2417 | C ++ |
| Shortest | BelarusianSU 5 | 1 | 269 | 1230 | C ++ |
| Max atts. | KBTU 1 | 8 | 248 | 2372 | C ++ |

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- Each tree has at least one centroid, such node that its removal splits the tree in connected components of size no more than $\frac{n}{2}$.
- Thus, we can pick this centroid and the jury pics one the components.
- The process will finish in no more than $\log n+1$ steps.


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- We maintain a set of candidates $A$. Initially it contains all nodes of given graph $G$.


## Problem C. Cactus Search

- Now we consider cactus case.
- Actually, ignore the fact we are given a cactus and solve the problem for any connected graph.
- Apply Floyd algorithm to compute a matrix of distances $\rho(u, v)$.
- We maintain a set of candidates $A$. Initially it contains all nodes of given graph $G$.
- If we ask about node $u$ and node $v$ is given as a reply we eliminate from set $A$ all nodes $w \in A$ such that $\rho(v, w)<\rho(u, w)$.


## Problem C．Cactus Search

－We aim to eliminate as many nodes from $A$ as possible．

## Problem C. Cactus Search

- We aim to eliminate as many nodes from $A$ as possible.
- Consider every node $v$ (not necessaritly in $A$ ). For each $u$ neighbor of $v$ compute the number of nodes $w$ in $A$ that are closer to $u$.


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- Pick $v$ that has minimum maximum number of such $w$ 's among all $u$ 's.


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- How many steps it will take to reduce $A$ down to a single node?


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- Pick $v$ that has minimum maximum number of such w's among all $u$ 's.
- How many steps it will take to reduce $A$ down to a single node?
- We claim there always exists such $v$ that the size of $A$ will reduce at least twice.


## Problem C. Cactus Search

- Let $P(v)=\sum_{u \in A} \rho(v, u)$. If more than a half of nodes $w \in A$ are closer to some $u$ (neighbor of $v$ ), then $P(u)<P(v)$.


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- The number of steps is $\log n+1$ and the running time is $O\left(n^{2}\right)$ per single query.


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- If we pick the node that minimizes $P(V)$ it will eliminate at least a half of $A$.
- The number of steps is $\log n+1$ and the running time is $O\left(n^{2}\right)$ per single query.
- That results in $O\left(n^{3}\right)$ per test case.


## Problem D. Distance Sum

Author: Gennady Korotkevich

Statements and tests: Gennady Korotkevich


|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 0 | 0 | 0 |
| $\square$ Rejected | 2 | 0 | 8 | 9 | 19 |
| Total | 2 | 0 | 8 | 9 | 19 |

## Problem D. Distance Sum

- Need: $\sum_{u=1}^{n-1} \sum_{v=u+1}^{n} d(u, v)$


## Problem D．Distance Sum

－Need：$\sum_{u=1}^{n-1} \sum_{v=u+1}^{n} d(u, v)$
－Generalize！Vertex $i \rightarrow$ weight $w_{i}$
－Need：$\sum_{u=1}^{n-1} \sum_{v=u+1}^{n} w_{u} w_{v} d(u, v)$

## Problem D. Distance Sum

- While there is a vertex of degree 1 :
- Pick any such vertex $v$, let its only neighbor be $u$
- Add $w_{v} \cdot\left(n-w_{v}\right)$ to the answer
- Increase $w_{u}$ by $w_{v}$
- Remove v


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- If there is a single vertex of degree 0 , exit


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- Increase $w_{u}$ by $w_{v}$
- Remove $v$
- If there is a single vertex of degree 0 , exit
- Otherwise all vertices have degree at least 2
- $\operatorname{deg}(v)>2 \Longrightarrow v$ is special
- $m \leq n+42 \Longrightarrow$ there are at most 84 special vertices


## Problem D. Distance Sum

- 84 special vertices connected by paths of non-special vertices
- Find the distance from every special vertex to all other vertices using BFS


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- 84 special vertices connected by paths of non-special vertices
- Find the distance from every special vertex to all other vertices using BFS
- $s(v)=\sum_{u \neq v} w_{u} d(u, v)$
- The answer is $\frac{1}{2} \sum_{v} s(v)$


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- How to calculate $s(v)$ ?


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- For special vertices $u, d(u, v)$ is already known


## Problem D. Distance Sum

- How to calculate $s(v)$ ?
- For special vertices $u, d(u, v)$ is already known
- Consider a non-special vertex $u$ lying on a path between special vertices $u_{1}$ and $u_{2}$ :
- $u_{1}, x_{1}, x_{2}, \ldots, x_{k}, u_{2}$


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- Consider a non-special vertex $u$ lying on a path between special vertices $u_{1}$ and $u_{2}$ :
- $u_{1}, x_{1}, x_{2}, \ldots, x_{k}, u_{2}$
- The shortest path from $v$ to $u$ visits either $u_{1}$ or $u_{2}$
- $\exists t \in[0 ; k]$ : the shortest path from $v$ to $x_{1}, x_{2}, \ldots x_{t}$ visits $u_{1}$, and the shortest path from $v$ to $x_{t+1}, x_{t+2}, \ldots, x_{k}$ visits $u_{2}$


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- $t$ can be calculated based on $d\left(v, u_{1}\right), d\left(v, u_{2}\right)$, and $k$


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- $u_{1}, x_{1}, x_{2}, \ldots, x_{k}, u_{2}$
- The shortest path from $v$ to $u$ visits either $u_{1}$ or $u_{2}$
- $\exists t \in[0 ; k]$ : the shortest path from $v$ to $x_{1}, x_{2}, \ldots x_{t}$ visits $u_{1}$, and the shortest path from $v$ to $x_{t+1}, x_{t+2}, \ldots, x_{k}$ visits $u_{2}$
- $t$ can be calculated based on $d\left(v, u_{1}\right), d\left(v, u_{2}\right)$, and $k$
- The part of $s(v)$ dependent on $x_{1}, x_{2}, \ldots, x_{k}$ can be calculated based on prefix sums of $w_{x_{1}}, w_{x_{2}}, \ldots, w_{x_{k}}$ and $1 \cdot w_{x_{1}}, 2 \cdot w_{x_{2}}, \ldots, k \cdot w_{x_{k}}$


## Problem E. Easy Chess

Author: Mikhail Dvorkin Statements and tests: Mikhail Dvorkin



|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 8 | 1 | 219 | 21 | 249 |
| $\square$ Rejected | 67 | 1 | 198 | 50 | 316 |
| Total | 75 | 2 | 417 | 71 | 565 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | MIPT 1 | 1 | 17 | 2237 | C++ |
| Shortest | Ataturk-Alatoo 2 | 2 | 167 | 417 | Python |
| Max atts. | IrkutskNRTU 1 | 54 | 293 | 3542 | Java |

## Problem E. Easy Chess

- Visited cells per column, $n=2:[1,0,0,0,0,0,0,2]$.
- Add +1 arbitrarily $n-2$ times, keeping $\leq 8$.


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- Visited cells per column, $n=2$ : $[1,0,0,0,0,0,0,2]$.
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- Process columns left to right. If visited[i] is 0: skip.
- Otherwise: enter at current row, arbitrarily visit some cells.


## Problem E. Easy Chess

- Visited cells per column, $n=2:[1,0,0,0,0,0,0,2]$.
- Add +1 arbitrarily $n-2$ times, keeping $\leq 8$.
- Process columns left to right. If visited [i] is 0 : skip.
- Otherwise: enter at current row, arbitrarily visit some cells.
- Constraints:
- in column a: start in row 1
- in columns a-g: don't finish in row 8
- in column h: finish in row 8


## Problem F. Fractions

Author: Dmitry Yakutov Statements and tests: Dmitry Yakutov


|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 143 | 5 | 148 |
| $\square$ Rejected | 4 | 2 | 512 | 11 | 529 |
| Total | 4 | 2 | 655 | 16 | 677 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | SPbSU 3 | 1 | 14 | 1224 | C++ |
| Shortest | CrimeanFU 1 | 1 | 123 | 381 | Python |
| Max atts. | RybinskSATU | 10 | 298 | 2515 | C++ |

## Problem F. Fractions

- If $n=p^{k}$, where $k>0$ and $p$ is prime, then print " NO ".
- Otherwise there exist such integers $a, b$ that $n=a \cdot b, 1<a, b<n$, $\operatorname{gcd}(a, b)=1, a \leq \sqrt{n}$. You can do it in $O(\sqrt{n})$.
- Let's try to represent $\frac{n-1}{n}$ as a sum of two fractions $\frac{n-1}{n}=\frac{x}{a}+\frac{y}{b}$.
- It is always possible to find such $x$ and $y(1 \leq x<a, 1 \leq y<b)$ that $\frac{n-1}{n}=\frac{x}{a}+\frac{y}{b}$.
- Just iterate over all possible values $x$ between 1 and $a-1$ and check that $y=\frac{n-1-x \cdot b}{a}$ is integer. It requires $O(\sqrt{n})$ steps.
- Total time complexity is $O(\sqrt{n})$.


## Problem F. Fractions: Proof

- For each $x$ between 1 and $a-1$ the value $y=\frac{n-1-x \cdot b}{a}>0$ (since $a \cdot b=n$ ).
- Show that $y=\frac{n-1-x \cdot b}{a}$ is integer for some $x$ between 1 and $a-1$.
- Let's try all $x=0 \ldots a-1$ and look on $(n-1-x \cdot b) \bmod a$.
- For $x=0(n-1-0 \cdot b) \bmod a \neq 0$.
- If for all $x=1 \ldots a-1$ all values $(n-1-x \cdot b) \bmod a \neq 0$, then there are two equal remainder.
- Say, $\left(n-1-x_{1} \cdot b\right) \bmod a=\left(n-1-x_{2} \cdot b\right) \bmod a$.
- It means $\left(x_{1}-x_{2}\right) \cdot b \bmod a=0$, but it is imposible since $\left|x_{1}-x_{2}\right|<a$.


## Problem G. Guest Student

Author: Mikhail Mirzayanov
Statements and tests: Mikhail Mirzayanov



|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 4 | 1 | 208 | 12 | 225 |
| $\square$ Rejected | 25 | 0 | 324 | 15 | 364 |
| Total | 29 | 1 | 532 | 27 | 589 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | IvanovoSPowU | 1 | 14 | 810 | C++ |
| Shortest | YerevanSU 2 | 2 | 64 | 443 | Python |
| Max atts. | IrkutskSU 3 | 9 | 242 | 1456 | C++ |

## Problem G. Guest Student

- Try each day of a week to start education.
- For each fixed starting day of week solve the problem independently.
- Calculate number of whole weeks. Roughly speaking, number of whole weeks is $\approx k /\left(a_{1}+a_{2}+\cdots+a_{7}\right)$. Process the remainder day by day.
- Return the best result over all possible 7 starting days of a week.


## Problem H. Harder Satisfiability

Author: Andrey Stankevich

Statements and tests: Artem Vasilyev


|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 1 | 0 | 1 |
| $\square$ Rejected | 0 | 0 | 10 | 0 | 10 |
| Total | 0 | 0 | 11 | 0 | 11 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| The only | MSU 3 | 3 | 210 | 2318 | C ++ |

## Problem H. Harder Satisfiability

Recall a solution for classic 2-SAT problem:

- Convert formula to implication graph with vertices $x_{i}, \overline{x_{i}}$ for all variables: $x \vee y \Rightarrow(\bar{x} \rightarrow y),(\bar{y} \rightarrow x)$
- Important property: if $x$ is reachable from $y$, than $\bar{y}$ is reachable from $\bar{x}$.
- Find strong connected components (SCC) in this graph.
- If for any vertex $x$ and $\bar{x}$ in same component, no solution exists.
- Otherwise, assign values to variables using the topological order.


## Problem H. Harder Satisfiability

Necessary conditions:

- For all $i: x_{i}$ and $\overline{x_{i}}$ are not in same SCC.
- For all $i<j$ with $\exists$ quantifier near $x_{i}$ and $\forall$ near $x_{j}$ : none of $x_{i}, \overline{x_{i}}$ is in one SCC with $x_{j}, \overline{x_{j}}$.
- For all $i, j$ with $\forall$ quantifiers: none of $x_{i}, \overline{x_{i}}, x_{j}, \overline{x_{j}}$ are not reachable from each other.


## Problem H. Harder Satisfiability

And they are sufficient!

- For all $i: x_{i}$ and $\overline{x_{i}}$ not in same SCC.
- For all $i<j$ with $\exists$ quantifier near $x_{i}$ and $\forall$ near $x_{j}$ : none of $x_{i}, \overline{x_{i}}$ is in one SCC with $x_{j}, \overline{x_{j}}$.
- For all $i, j$ with $\forall$ quantifiers: none of $x_{i}, \overline{x_{i}}, x_{j}, \overline{x_{j}}$ are not reachable from each other.

Assign algorithm:

- Condition 2 allows us to mark SCC as "any", if it have a $\forall$ vertex inside.
- Let's make all components reachable from "any" as true, and their negations as false.
- This doesn't lead to a contradiction, because if some vertex is reachable from one forall and some forall is reachable from it, then the third condition fails.
- Other components can be decided the same way as classic 2-SAT


## Problem I. Interval-Free Permutations

Author: Andrey Stankevich
Statements and tests: Pavel Kunyavsky


|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 2 | 0 | 2 |
| $\square$ Rejected | 0 | 0 | 3 | 0 | 3 |
| Total | 0 | 0 | 5 | 0 | 5 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | MIPT 6 | 2 | 216 | 2063 | C ++ |
| Shortest | MSU 3 | 2 | 279 | 1468 | C ++ |
| Max atts. | MSU 3 | 2 | 279 | 1468 | C ++ |

## Problem I. Interval-Free Permutations

- Use dynamic programming. Count all permutations and subtract bad ones
- Let's call an interval a block, if it's not inside any other interval, except for an entire permutation.
- Two blocks either non-intersecting or cover full permutation


## Problem I. Interval-Free Permutations

- Permutation is either split by at least three blocks or covered by two.
- In the first case, splitting is unique. We can choose any permutation in each block, while permutation of blocks should be good.
- In the second case, there are several ways to split. To make it unique, we need to force left permutation have no prefixes, which are permutation.


## Problem I. Interval-Free Permutations

- Number of permutation, none of prefixes of which is permutation.

$$
I_{n}=n!-\sum_{k=1}^{n-1} I_{k} \cdot(n-k)!
$$

- Number of ways to choose $k$ permutations of total length $n$.

$$
B_{n, k}=\sum_{t=1}^{n} B_{n-t, k-1} \cdot t!
$$

- The answer

$$
A_{n}=n!-2 \cdot \sum_{k=1}^{n-1} I_{k} \cdot(n-k)!-\sum_{k=3}^{n-1} B_{n, k} \cdot A_{k}
$$

## Problem J. JS Minification

## Author: Roman Elizarov <br> Statements and tests: Roman Elizarov



|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 5 | 0 | 5 |
| $\square$ Rejected | 0 | 0 | 44 | 1 | 45 |
| Total | 0 | 0 | 49 | 1 | 50 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | MIPT 6 | 2 | 203 | 7724 | C ++ |
| Shortest | MSU 3 | 1 | 242 | 4493 | C ++ |
| Max atts. | ITMO 4 | 5 | 261 | 6512 | C ++ |

## Problem J. JS Minification

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- Either longest word/number or a reserved token


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- Parse input
- Either longest word/number or a reserved token
- Rename words
- Remember to skip reserved
- Write output
- Greedily insert spaces when needed
- Keep a list of tokens written since the last space


## Problem K. King Kog's Reception

Author: Vitaliy Aksenov

Statements and tests: Vitaliy Aksenov


|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 0 | 0 | 20 | 0 | 20 |
| $\square$ Rejected | 0 | 0 | 45 | 0 | 45 |
| Total | 0 | 0 | 65 | 0 | 65 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | MIPT 6 | 1 | 79 | 3191 | $\mathrm{C}++$ |
| Shortest | SPbHSE 1 | 2 | 250 | 2168 | $\mathrm{C}++$ |
| Max atts. | UofLatvia 1 | 5 | 219 | 3819 | $\mathrm{C}++$ |

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\max _{i: t_{i} \leq T}\left(t_{i}+\sum_{j: t_{i} \leq t_{j} \leq T} d_{j}\right)
$$

- (Knight $i$ comes in his time, all next knights wait consecutively.)


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- Right part: interval tree with sum
- Left part: adding/removing a knight affect a segment.
- So, interval tree with addition on a segment.


## Problem L. Lazyland

Author: Pavel Mavrin
Statements and tests: Pavel Mavrin



|  | Java | Kotlin | C ++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 7 | 1 | 226 | 13 | 247 |
| $\square$ Rejected | 14 | 0 | 204 | 25 | 243 |
| Total | 21 | 1 | 430 | 38 | 490 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | BelarusianSU 1 | 1 | 7 | 965 | C++ |
| Shortest | SUrSU 3 | 1 | 30 | 367 | Python |
| Max atts. | RybinskSATU | 8 | 121 | 955 | C++ |

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## Problem L. Lazyland

- For each job, find the idler with the maximum value of $b_{i}$.
- Assign this job to this idler.
- Put all remaining idlers in the array, sort them by the value of $b_{i}$.
- Assign idlers with minimum values to remaining jobs.


## Problem M. Minegraphed

Author: Mikhail Dvorkin
Statements and tests: Mikhail Dvorkin


|  | Java | Kotlin | C++ | Python | Total |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\square$ Accepted | 1 | 0 | 65 | 0 | 66 |
| $\square$ Rejected | 6 | 0 | 206 | 0 | 212 |
| Total | 7 | 0 | 271 | 0 | 278 |


| solution | team | att | time | size | lang |
| :--- | :--- | ---: | ---: | ---: | :--- |
| Fastest | ITMO 1 | 1 | 67 | 2304 | C ++ |
| Shortest | UrFU 7 | 3 | 254 | 1844 | C ++ |
| Max atts. | TbilisilBSU 1 | 9 | 255 | 2286 | C++ |

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- For each $i$, make 2-step "staircase" from bottom to top.


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- Vertex $i$ : north-south tunnel in bottom layer + west-east tunnel in top layer.
- Full-length tunnels with 2 walls between: $\{x=3 i \wedge z=0\} \cup\{y=3 i \wedge z=2\}$.
- For each $i$, make 2-step "staircase" from bottom to top.
- For each edge $i \rightarrow j$, make vertical hole to fall from $i$-th top tunnel to $j$-th bottom tunnel.


## Credits

- Special thanks to all jury members and assistants (in alphabetic order):

Andery Halyavin, Andrey Stankevich, Artem Vasilyev, Borys Minaiev, Dmitry Yakutov, Evgeniy Kuprilyanskiy, Gennady Korotkevich, Gleb Evstropov, Ilya Zban, Ivan Belonogov, Ivan Kazmenko, Mikhail Dvorkin, Mikhail Mirzayanov, Mikhail Tikhomirov, Niyaz Nigmatullin, Oleg Hristenko, Pavel Irzhavski, Pavel Kunyavskiy, Pavel Mavrin, Roman Elizarov, Vitaly Aksenov

