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Define sequences p and q such that $p_i = max(a_0, a_1, \dots, a_i)$, and $q_i = max(a_i, a_{i+1}, \dots, a_{n-1})$

Observe that the amount of accumulating rain is $\sum_{i=0}^{n-1} min(p_i, q_i) - a_i$. It's easy to update $\sum_i a_i$ based on the range increase operation, so let's focus on $\sum_{i=0}^{n-1} min(p_i, q_i)$.

A critical observation is that p_i is (non-strictly) increasing function, while q_i is (non-strictly) decreasing function. Another critical observation is that operation $a[l..r]$`+=1` only causes at most one range update for p: $p[l'..r']$`+=1` (if p would change at all, the first element to increase would be the leftmost element on segment $[l; r]$ larger than $max(a_0, \dots, a_{l-1})$, and this increase will go until first sufficiently large element to the right of it. Both indices can be found with standard segment tree algorithms in $O(\log n)$ time). Similarly there at most one range update $q[l''..r']$`+=1`.

There are some different ways how to finalize calculation give those ideas. For instance, one can store a segment tree for $p_i - q_i$ (observe this is monotonically increasing function), and then at each `p[l..r] +=1` and `q[l..r]+=1` range operations caused by above, recompute how $\sum_{i=0}^{n-1} min(p_i, q_i)$ is changing. Specifically, $min(p_i, q_i)$ will increase during `p[l..r]+=1` operation for indices i such that $p_i < q_i$.